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Adaptive Estimation Techniques for Hidden Markov Models

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of the Australian National University*

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Research School of Physical Sciences
and
Engineering
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Declaration

These doctoral studies were conducted with Professor John B. Moore as supervisor, and Professor Brian D.O. Anderson and Dr. Robert R. Bitmead as advisors.

The contents of this thesis are the results of original research, and have not been submitted for a higher degree at any other university or institution.



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- ML techniques for extracting Markov signals imbedded in a mixture of white Gaussian noise and deterministic disturbances of known functional form with unknown parameters. Two such disturbances are considered: periodic disturbances and polynomial drift in the Markov states.
- Adaptive on-line schemes for estimating time-varying HMMs and Hidden semi-Markov models. We also propose on-line schemes for adaptively extracting Markov signals with time varying statistics imbedded in a mixture of white Gaussian noise and deterministic disturbances with time-varying parameters. In contrast to the off-line estimation techniques mentioned above, the on-line schemes can adaptively learn time-varying models. Also the memory and computational requirements are significantly reduced compared to off-line processing.

The discrete-state techniques have applications which we have explored in neurobiological signal processing; in particular, for the extraction of channel currents from noisy measurements in the ionic channels of cell membranes. Extensive simulation studies have been carried out to confirm the robustness of the proposed algorithms. The HFM processing schemes and extraction schemes in the presence of deterministic interferences have also been applied to experimental data obtained from noisy measurements of channel currents in cell membranes.

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Preface

This thesis is organized into 8 chapters.

Chapter 1 introduces the basic concepts and presents an overview of the thesis.

Chapters 2 and 3 discuss ARMAX estimation schemes.

Chapters 4, 5, 6 and 7 deal with the estimation of discrete-state Hidden Markov Models.

Chapter 8 summarizes the contributions of the thesis and highlights directions for future research.

The equations, sections and figures are numbered according to the chapter in which they occur. For ready reference, a glossary of symbols and commonly used abbreviations is provided following the Contents. The references are arranged in order of appearance and are listed in the Bibliography.

Most of the work reported in this thesis has already been published or submitted for publication in technical journals. A list is given below:

- L.Xia, V. Krishnamurthy and J.B Moore, *Adaptive Estimation in the presence of Order and Parameter changes*, Int. Journal of Adaptive Control and Signal Processing, Vol 3, No.3, pp 283-292, Sept. 1989.
- V. Krishnamurthy, B. Wahlberg and J.B. Moore, *Factorizations that relax the Positive Real Condition in Continuous-time and Fast-Sampled ELS Schemes*, Int. Journal of Adaptive Control and Signal Processing, Vol 4, No.5, pp 389-414, Sept 1990.
- V. Krishnamurthy, J.B. Moore and S.H. Chung, *Hidden Fractal Model Signal Processing*, Signal Processing, Vol 24, 1991 (to appear).
- S.H. Chung, V. Krishnamurthy, J.B. Moore, *Adaptive Processing techniques based on Hidden Markov Models for characterizing very small channel currents buried in noise and deterministic interferences*, Philosophical Transactions of the Royal

Society, (accepted for publication).

- V. Krishnamurthy, J.B. Moore, *Discrete semi-Markov Signal Processing with Decaying States*, Signal Processing, (submitted for publication).
- V. Krishnamurthy, J.B. Moore, S.H. Chung, *Hidden Markov Model Signal Processing in the presence of unknown deterministic interferences*, IEEE Trans. Auto. Control, (submitted for publication).
- V. Krishnamurthy, J.B. Moore, *On-line Hidden Markov Model Signal Processing and Elimination of deterministic interferences*, IEEE Trans. Signal Processing, (submitted for publication).

Papers submitted in Conference Proceedings, but in some cases containing material overlapping with the above are:

- B. Wahlberg, V. Krishnamurthy, J.B. Moore, *Factorizations that relax the Positive Real Condition in continuous-time ELS schemes*, IFAC World Congress, Tallinn, pp 209-214, Aug. 1990.
- V. Krishnamurthy, J.B. Moore and L. Xia, *Towards On-line Hidden Markov Model Signal Processing*, Proc. of 29th IEEE Conference on Decision and Control, pp 815-810, Hawaii, Dec. 1990.
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- V. Krishnamurthy, J.B. Moore, *Elimination of sinusoidal disturbances using HMM processing*, 30th IEEE Conference on Decision and Control, Brighton, U.K., 1991, (accepted for publication).

- V. Krishnamurthy, J.B. Moore, *Discrete Hidden semi-Markov Model signal processing with decaying states*, 30th IEEE Conference on Decision and Control, Brighton, U.K., 1991, (accepted for publication).

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Glossary of frequently used symbols and abbreviations

Continuous-state Hidden Markov Models:

ARMAX: Autoregressive Moving Average systems with Exogenous Inputs

ELS : Extended Least Squares

KF : Kalman Filter

OE systems : Output Error systems

p^{-1} : Differential operator; also denoted as s^{-1}

PE : Persistently Exciting

PKF : Perturbed Kalman Filter

SBR : Strictly Bound Real

SPR : Strictly Positive Real

q^{-1} : Delay operator; also denoted as z^{-1}

Discrete-state Hidden Markov Models:

A : Transition probability matrix

b : Symbol probability matrix

EM algorithm : Expectation Maximization algorithm

HMM : Hidden Markov Model

MAP estimate : Maximum a posteriori estimate

ML : Maximum Likelihood

s_k : Finite-state discrete-time Markov process

WGN : White Gaussian Noise

$\mathbf{Y}_k$: Observation sequence $y_1, \dots, y_k$

$\underline{\pi}$: Initial state probability vector

σ_w : Standard deviation of white noise